# General Mathematics and Computational Science I

Midterm I

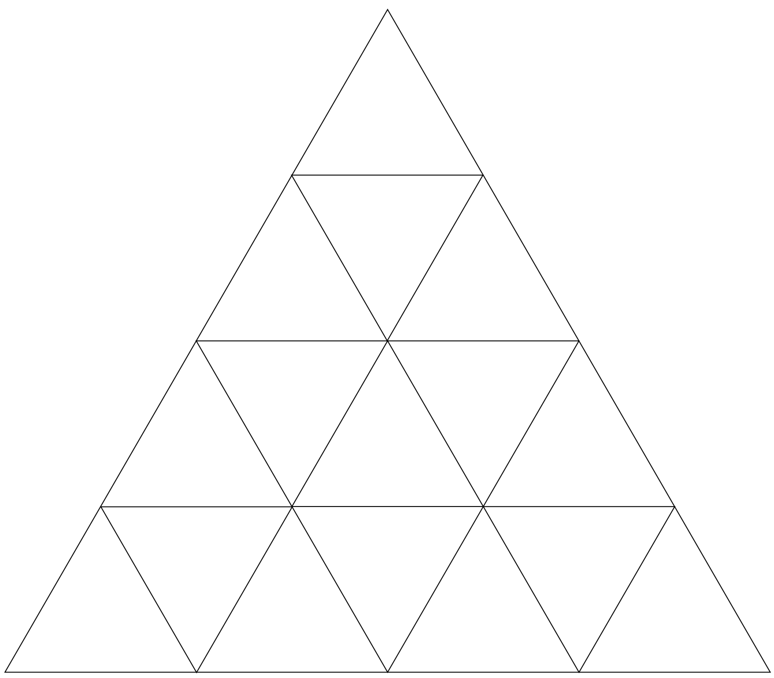
October 5, 2006

1. Prove by induction that

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{n \cdot (n+1)} = \frac{n}{n+1}$$

   for every $n \in \mathbb{N}$. (8)

2. A triangle is divided by lines parallel to each of its sides into $T_n$ smaller triangles. (The figure below shows the case $n = 4$ where $T_n = 16$.) Find a formula for $T_n$ and prove that your formula is correct.



(8)

3. Give one example each of a function $f \colon \mathbb{N} \to \mathbb{N}$ that is

(a) bijective,

(b) injective but not surjective,

(c) surjective but not injective,

(d) neither surjective nor injective.

(2+2+2+2)

4. Consider the set $Q = \{(a, b) \colon a, b \in \mathbb{Z} \text{ and } b \neq 0\}$. Let $(a, b) \sim (c, d)$ if and only if $ad = bc$.

   (a) Show that $\sim$ is an equivalence relation, i.e. that it is reflexive, symmetric, and transitive.

   (b) Define an operation $\circ$ on $Q$ via $(a, b) \circ (c, d) = (ad + bc, bd)$. Show that $\circ$ is well defined on classes $[a, b]$ with respect to the equivalence relation $\sim$. In other words, prove that if $(a, b) \sim (a', b')$, then $(a, b) \circ (c, d) \sim (a', b') \circ (c, d)$.

(6+4)

5. Consider a map $G \colon \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ with the following properties:

   (M1) $G(a, 1) = a$ for all $a \in \mathbb{N}$,

   (M2) $G(a, s(b)) = G(a, b) + a$ for all $a, b \in \mathbb{N}$.

   Prove, by showing that a certain set is inductive, that $G(a, c) = G(b, c)$ implies $a = b$ for any $a, b, c \in \mathbb{N}$. (8)

6. Show that $\mathbb{N} \cong \{n \in \mathbb{N} : n \text{ even}\}$. (8)