General Mathematics and Computational Science I

Exercise 16

November 23, 2006

1. The population of some species is modeled by

$$x_{n+1} = \mu\, x_n\, \mathrm{e}^{-x_n}\,, \tag{1}$$

with $\mu > 0$.

   (a) Prove that $x^* = \ln \mu$ is the only equilibrium point.

   (b) For which values of μ is the equilibrium point asymptotically stable, for which values is it unstable?

2. For a given continuously differentiable function g, consider the difference equation

$$x_{n+1} = x_n + h\, g(x_n)\,.$$

Determine conditions on h for which an equqilibrium point is asymptotically stable, respectively unstable.

Remark: This difference equation is in fact the so-called Euler method for solving the differential equation $\dot{y} = g(y)$. However, this is not relevant for answering this question.

3. Show that the logistic map

$$x_{n+1} = \mu\, x_n\, (1 - x_n)$$

has a 2-cycle whenever $\mu > 3$ with

$$a = \frac{1 + \mu - \sqrt{(\mu - 3)(\mu + 1)}}{2\,\mu}\,,$$
$$b = \frac{1 + \mu + \sqrt{(\mu - 3)(\mu + 1)}}{2\,\mu}\,.$$