

General Mathematics and Computational Science I

Exercise 15

November 21, 2006

1. (*Extension from Homework Set 14.*) Use Laplace's method to find the leading term in the asymptotic behavior of

$$\int_{-1}^1 e^{-s \cosh x} dx$$

as $s \rightarrow \infty$.

2. (From Elaydi, p. 28.) Find the equilibrium points of the difference equation

$$x_{n+1} = \frac{1}{2} x_n^3 + \frac{1}{2} x_n$$

and determine their stability.

3. (From Elaydi, p. 34.) Suppose that the difference equation

$$x_{n+1} = f(x_n)$$

has a 2-cycle whose orbit is $\{a, b\}$. Show that

- (a) the 2-cycle is asymptotically stable if $|f'(a) f'(b)| < 1$,
- (b) the 2-cycle is unstable if $|f'(a) f'(b)| > 1$.

Hint: Note that a 2-cycle is an equilibrium point of $y_{n+1} = f(f(y_n))$.