General Mathematics and Computational Science I

Exercise 14

November 16, 2006

1. Check the conditions for Laplace's method for the function

$$f(x) = x - \ln x - 1$$

which appears in the proof of Stirling's formula. I.e., check the following.

(a) f is strictly decreasing for $0 < x < 1$, strictly increasing for $x > 1$, and $f(1) = 0$.

(b) There are positive constants b and c such that $f(x) \geq b\,x$ for $x \geq c$.

(c) $f(x) = a\,(x-1)^2 + \psi(x)\,(x-1)^3$ where ψ is a bounded function for $x \in [1-\delta, 1+\delta]$ for some $\delta > 0$. How must a be chosen?

Hint: For (c), use either Taylor's formula with remainder (easy), or l'Hôpital's rule for the function

$$g(x) = \frac{f(x)}{(x-1)^2}\,.$$

(more elementary, but longer).

2. Use Laplace's method to find the leading term in the asymptotic behavior of

$$\int_{-1}^{1} \mathrm{e}^{-s \cosh x}\,\mathrm{d}x$$

as $s \to \infty$.

3. A loan of L Euros is to be amortized by equal monthly payments. The yearly interest rate is r, compounded monthly. Derive a formula for the monthly payment required to pay off the loan in T years.