General Mathematics and Computational Science I

Exercise 8

October 19, 2006

1. Show that

$$\sum_{k=0}^{n} \binom{n+k}{k} \frac{1}{2^k} = 2^n$$

   Hint: Denote the left hand side by $f(n)$ and prove that $f(n + 1) = 2 f(n)$. Use Exercise 6, Problem 2.

2. Show that

$$\binom{n+1}{k+1} = \sum_{j=k}^{n} \binom{j}{k}$$

   for every $0 \le k \le n$.

3. Use the method of generating functions to find a closed form expression for the members of the generalized Fibonacci sequence

$$a_0 = A \, ,$$
$$a_1 = B \, ,$$
$$a_n = a_{n-1} + a_{n-2} \, .$$