

# General Mathematics and Computational Science I

## Exercise 6

October 12, 2006

1. A company has  $2n$  employees.
  - (a) In how many ways can they be divided into  $n$  groups of two members each?
  - (b) In how many ways can they be divided into two departments of  $n$  members each, if each department must choose a president and a vice-president?
2. (From Ivanov, p. 17.)
  - (a) Verify, by explicit computation, that the binomial coefficients

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

satisfy the recursion relation

$$\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}. \quad (*)$$

- (b) Derive this recursion relation from the interpretation of the binomial coefficients as the “number of  $k$ -element subsets of an  $n$ -element set”.
3. (From Ivanov, p. 19.) Prove that

$$\binom{2n}{k} = \sum_{l=0}^k \binom{n}{l} \binom{n}{k-l}.$$

Hint: Use the function  $P_n(x)$  from class, and the fact that

$$((1+x)^n)^2 = (1+x)^{2n}.$$