General Mathematics and Computational Science I

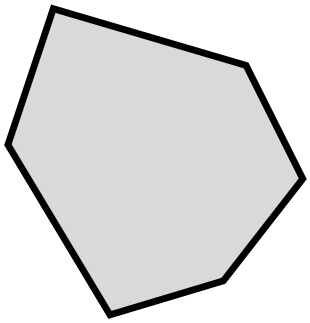
Exercise 1

September 14, 2006

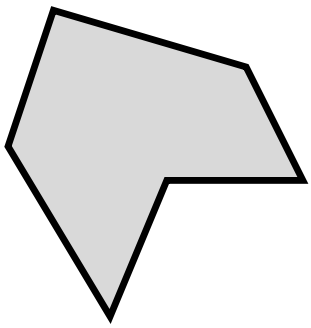
1. (From Ivanov, p. 2) Into how many pieces do
   (a) n points subdivide a line;
   (b) n straight lines subdivide the plane, if no two of the lines are parallel and no three meet in a single point?

2. Use induction to show that for $n \in \mathbb{N} \cup \{0\}$ and any (real) number $q \neq 1$,
$$\sum_{k=0}^{n} q^k = \frac{1 - q^{n+1}}{1 - q}\,.$$
   Remark: This sum is called a *geometric progression*.

3. A polygon is called *convex* if any straight line connecting two points of the polygon lies entirely within; see figure below.
   Use induction to show that, for a convex n-gon (a polygon with n vertices), the sum of the interior angles S_n is
$$S_n = 180 \cdot (n - 2)$$



Convex polygon



Non−convex polygon

4. (From Daepp/Gorkin, p. 114) Let X be a nonempty set with an equivalence relation $\sim$ on it. Prove that for all $x, y \in X$,
$$[x] = [y] \qquad \text{if and only if} \qquad x \sim y\,.$$