Numerical Methods I – Lab 5

Fall Semester 2005

October 25, 2005

1. It can be shown that the error of approximating a function f via a polynomial p_n of degree less or equal to n can be estimated by

$$|f(x) - p_n(x)| \leq \frac{M_{n+1}}{(n+1)!} \left|\pi_{n+1}(x)\right| ,$$

where

$$M_{n+1} = \max_{x \in [x_0, x_n]} \left|f^{(n+1)}(x)\right| ,$$

$$\pi_{n+1}(x) = \prod_{j=1}^{n} (x - x_j) ,$$

and where $x_0, \ldots, x_n$ are $n+1$ distinct interpolation nodes.

In other words, the error is determined by two factors: One depends only on the function f and will not be considered here. The other factor depends only on the distribution of nodes.

(a) Plot $\pi_{n+1}(x)$ for an equidistantly distributed nodes x_0,...,x_n on the interval $[-5, 5]$ for $n = 10, 20$.

(b) Plot $\pi_{n+1}(x)$ for the so-called Chebycheff-nodes

$$x_i = 5 \cos \frac{i\,\pi}{n}$$

where $i = 0, \ldots, n$.

(c) Plot both graphs into the same plot, and label the graphs.

Hints: `help plot` for labeling, `help poly` and `help polyval` for the quick construction and evaluation of polynomials.

2. The polynomial interpolation problem, i.e. finding

$$p_n(x) = c_0 + c_1\,x + \cdots + c_n\,x^n$$

such that $p_n(x_i) = y_i$ for $i = 0, \ldots, n$, leads immediately to the system of linear equations for the coefficients $c_0, \ldots, c_n$:

$$\begin{pmatrix} x_0^n & \cdots & x_0^0 \\ \vdots & & \vdots \\ x_n^n & \cdots & x_n^0 \end{pmatrix} \begin{pmatrix} c_n \\ \vdots \\ c_0 \end{pmatrix} = \begin{pmatrix} y_0 \\ \vdots \\ y_n \end{pmatrix}.$$

The matrix on the left is called *Vandermonde matrix*. (The Vandermonde matrix we have encountered in a previous example is the special case when the nodes are $1, \ldots, n$.)

(a) Construct the Vandermonde matrix for the two distributions of nodes from part 1.

(b) Use this approach to solve an interpolation problem, e.g. the problem from Homework 6 where

$$f(x) = \frac{1}{1+x^2}$$

on the interval $[-5, 5]$, and $y_i = f(x_i)$ for $i = 0, \ldots, n$.

Octave Trick: To generate an Encapsulated Postscript (EPS) file from a plot for printing or inclusion into other documents, use the following code.

```
gset term postscript eps monochrome; % omit 'monochome' for color output
gset output "filename.eps";          % change 'filename' as needed
replot;
gset term x11;                       % restore screen output if necessary
```

Note for Windows: The last line may work differently on Windows (any information is welcome). Also, you may need to install *ghostscript* and *gsview* in order to print and process Postscript files.