General Mathematics and Computational Science I

Practice Midterm II – Not for Credit

October 27, 2005

1. A certain door lock has five buttons. There is one sequence of buttons which opens the door.

    (a) How many combinations are possible if each button must be pressed exactly once?

    (b) Assume that there exists, in addition, sequences where one or more pairs of buttons must be pressed simultaneously. How many combinations are possible now?

2. Show that

$$\sum_{j=0}^{k} \binom{m}{j}\binom{n-m}{k-j} = \binom{n}{k}$$

    for integers $0 \le k \le m \le n$.

    (Use the generating function for the binomial coefficients.)

3. Solve the "hat check problem" (see separate handout).

4. Solve the recurrence relation

$$a_n - 5\,a_{n-1} + 6\,a_{n-2} = 0$$

    with $a_0 = 2$ and $a_1 = 5$, using the method of generating functions.

5. Ivanov, p. 46, Problem 1. (Try a selection of subproblems.)

6. Use the n-term arithmetic-geometric-mean inequality to derive Bernoulli's inequality

$$(1+x)^n \ge 1 + n\,x$$

    for $x \ge 0$ and n a natural number.