General Mathematics and Computational Science I

Practice Midterm I – Not for Credit

September 22, 2005

1. (Re)do Exercise 2 Question 3.
2. Show that $2^n \leq n!$ for all natural numbers $n \geq 5$.
3. Are the following functions surjective? Are they injective? Prove or disprove!
   (a) Define $f\colon \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ by $f(x, y) = (y, x)$.
   (b) Let X be a set, and $P(X)$ the set of all subsets of X, called the *power set* of X. Fix a proper subset $B \subset X$, and let $f\colon P(X) \to P(X)$ be defined as $f(A) = A \cap B$.
   (c) Define $f\colon P(\mathbb{N}) \to \mathbb{N}$ by $f(A) = \min A$, the minimum element of the set A.
4. Consider a map $G\colon \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ with the following properties:
   (M1) $G(a, 1) = a$ for all $a \in \mathbb{N}$,
   (M2) $G(a, s(b)) = G(a, b) + a$ for all $a, b \in \mathbb{N}$,

   where $s\colon \mathbb{N} \to \mathbb{N}$ is as in Peano's axioms.

   Prove that if
   $$G(a, c) = G(b, c)$$
   for some $a, b, c \in \mathbb{N}$, then $a = b$.
5. For functions $p, q\colon \mathbb{Z} \to \mathbb{Z}$, define the relation $p \sim q$ if and only if $p(0) = q(0)$.

   Is this an equivalence relation? Prove or disprove!
6. Let $I_n = \{k \in \mathbb{N}\colon k \leq n\}$. Show that
   $$I_m \times I_n \cong I_{mn}\,.$$