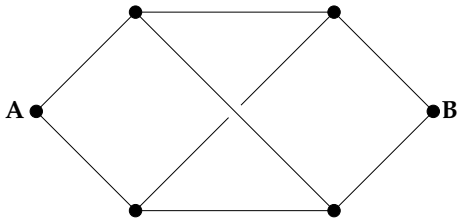
General Mathematics and Computational Science I

Midterm II

November 3, 2005

1. Cities A and B are connected as follows:



   (a) In how many ways can you travel from A to B without taking any road segment twice? Explain.

   (b) In how many ways can you travel from A to B and back without taking any road segment twice? Explain.

   (5+5)

2. Prove, using generating functions or otherwise, that

$$\sum_{k=1}^{n} k \binom{n}{k} = n\, 2^{n-1}\,.$$

   (10)

3. Four people get into an elevator in a six story building. What is the probability that they all get off at different floors? (Assume that each floor is equally likely to be visited.) (10)

4. Solve the recurrence relation

$$a_{n+1} = 3\, a_n - 2\, a_{n-1}$$

   with $a_0 = 3$ and $a_1 = 5$, using the method of generating functions. (10)

5. Show that, for a fixed perimeter, the rectangle with the largest area is the square. (10)

6. Let $a_1, \ldots, a_n$ be positive. Show that

$$\frac{a_1}{a_2} + \frac{a_2}{a_3} + \cdots + \frac{a_n}{a_1} \geq n\,.$$