General Mathematics and Computational Science I

Exercise 8

October 6, 2005

1. (From Ivanov, p. 17.)

   (a) Verify, by explicit computation, that the binomial coefficients

   $$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

   satisfy the recursion relation

   $$\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}\,. \qquad (*)$$

   (b) Derive this recursion relation from the interpretation of the binomial coefficients as the "number of k-element subsets of an n-element set".

2. (From Ivanov, p. 18.) Prove the binomial theorem directly by noting that the coefficients of the monomials $a^k\, b^{n-k}$ in the expansion of $(a+b)^n$ satisfy the recursion relation (*).

3. (From Ivanov, p. 19.) Prove that

   $$\binom{2n}{k} = \sum_{l=0}^{k} \binom{n}{l} \binom{n}{k-l}\,.$$

   Hint: Use the function $P_n(x)$ from class, and the fact that

   $$\bigl((1+x)^n\bigr)^2 = (1+x)^{2n}\,.$$