Partial Differential Equations

Homework 1

due September 14, 2004

1. Evans, p. 85 problem 1
2. Evans, p. 85 problem 2
3. Consider a function of one complex variable $w\colon \mathbb{C} \to \mathbb{C}$ on an open connected subset of the complex plane, and write $w = w(z)$ with $w = u + \mathrm{i}v$ and $z = x + \mathrm{i}y$. The function w is called *(complex) differentiable* or *holomorphic* if u and v have continuous first partial derivatives with respect to x and y that satisfy the so-called *Cauchy–Riemann equations*

   $$\begin{aligned} u_x &= v_y \\ u_y &= -v_x \end{aligned}$$

   It is known that holomorphic functions are infinitely differentiable.

   Show that the real and imaginary parts of a holomorphic function are harmonic.
4. Let $U \subset \mathbb{R}^n$ be open, and $u \in C^1(U)$. Show that

   $$\frac{u(x + h\, e_i) - u(x)}{h} \to \frac{\partial u}{\partial x_i}(x)$$

   uniformly on compact subsets of U as $h \to 0$.