

Solutions for Homework 10

3. (a) Critical points:

$$x(y-1) = 0 \Rightarrow x=0 \text{ or } y=1$$

$$y(1-x) = 0 \Rightarrow y=0 \text{ or } 1=x$$

So there are 2 critical points, $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

$$(b) \quad DF = \begin{pmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{pmatrix} = \begin{pmatrix} y-1 & x \\ -y & 1-x \end{pmatrix}$$

$$(c) \quad DF \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

\Rightarrow The eigenvalues are -1 and 1 ,
the critical point is unstable (is a saddle point)

$$DF \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

\Rightarrow The eigenvalues are i and $-i$,
the critical point is linearly neutrally stable (a center)

Phase portrait: (qualitatively)

