Numerical Methods I

Lab Session 9

November 27, 2003

Solve the following systems of differential equations, for example by using a fourth order explicit Runge–Kutta method,

$$\begin{aligned}
\boldsymbol{k}_1 &= \boldsymbol{f}(t_n, \boldsymbol{y}_n)\,, \\
\boldsymbol{k}_2 &= \boldsymbol{f}(t_n + \tfrac{1}{2}h, \boldsymbol{y}_n + \tfrac{1}{2}h\,\boldsymbol{k}_1)\,, \\
\boldsymbol{k}_3 &= \boldsymbol{f}(t_n + \tfrac{1}{2}h, \boldsymbol{y}_n + \tfrac{1}{2}h\,\boldsymbol{k}_2)\,, \\
\boldsymbol{k}_4 &= \boldsymbol{f}(t_{n+1}, \boldsymbol{y}_n + h\,\boldsymbol{k}_3)\,, \\
\boldsymbol{y}_{n+1} &= y_n + \tfrac{1}{6}h\,(\boldsymbol{k}_1 + 2\,\boldsymbol{k}_2 + 2\,\boldsymbol{k}_3 + \boldsymbol{k}_4)\,,
\end{aligned}$$

or any of the other methods from Homework 10.

1. The *Rössler system*,

$$\begin{aligned}
\dot{x} &= -y - z \\
\dot{y} &= x + \tfrac{1}{5}\,y \\
\dot{z} &= \tfrac{1}{5} + x\,z - \mu\,z
\end{aligned}$$

   with any initial condition. Solve the system over a long interval of time and plot the trajectory. What happens if you increase μ from 2 to 6?

   Hint: You can produce a parametric 3D plot as follows:

```
gset parametric
gsplot y'
```

2. The *Brusselator equation*,

$$\begin{aligned}
\dot{x} &= 1 - (b+1)\,x + a\,x^2\,y\,, \\
\dot{y} &= b\,x - a\,x^2\,y\,.
\end{aligned}$$

   Again, pick your initial data as you like, and study the behavior as $a = 1$ and b is increased from 1 to 3.