Numerical Methods I

Lab Session 8

November 20, 2003

The differential equation for the harmonic oscillator can be written in the form

$$\dot{\boldsymbol{y}} = A\boldsymbol{y}\,,$$
$$\boldsymbol{y}(0) = \boldsymbol{y}_0\,,$$

where $y\colon [0,T] \to \mathbb{R}^2$ and

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

Solve the harmonic oscillator with initial data

$$\boldsymbol{y}_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

on the interval $t \in [0, 100]$ with each of the following methods. Plot each of the solutions in the y_1-y_2 phase plane.

1. The *explicit Euler* method (i.e. the explicit Taylor method of order 1),

   $$\boldsymbol{y}_{n+1} = \boldsymbol{y}_n + h\,f(\boldsymbol{y}_n)\,,$$

   where, for the harmonic oscillator, $f(\boldsymbol{y}) = A\boldsymbol{y}$.

2. The *implicit Euler* method

   $$\boldsymbol{y}_{n+1} = \boldsymbol{y}_n + h\,f(\boldsymbol{y}_{n+1})\,.$$

3. The *implicit midpoint* method

   $$\boldsymbol{y}_{n+1} = \boldsymbol{y}_n + h\,f\Big(\frac{\boldsymbol{y}_n + \boldsymbol{y}_{n+1}}{2}\Big)\,.$$

4. The *symplectic Euler* method, which uses explicit Euler for the y_1 equation, and implicit Euler for the y_2 equation.