# Numerical Methods I

Lab Session 4

October 9, 2003

1. For given $n$, e.g. $n = 10$, construct the $n \times n$ matrix

$$A = \begin{pmatrix} 2 & -1 & 0 & & \cdots & & 0 & -\beta \\ -1 & 2 & \ddots & \ddots & & & & 0 \\ 0 & \ddots & \ddots & & & & & \\ & \ddots & & & & & & \vdots \\ \vdots & & & & & & \ddots & \\ & & & & & \ddots & \ddots & 0 \\ 0 & & & & \ddots & \ddots & 2 & -1 \\ -\beta & 0 & & \cdots & & 0 & -1 & 2 \end{pmatrix}$$

   and the vectors $\boldsymbol{b}, \boldsymbol{c} \in \mathbb{R}^n$ with

$$b_j = 1\,,$$
$$c_j = \sin \frac{2\pi j}{n}$$

   where $j = 1, \ldots, n$.

2. Use your code from Homework 4 to solve $A\boldsymbol{x} = \boldsymbol{b}$ and $A\boldsymbol{x} = \boldsymbol{c}$ for $\beta = 1/2$ and $\beta = 1$ via $LU$ decomposition.

3. Use the Gauss–Seidel iterative scheme to solve both problems. Does the solution converge?

4. Can you explain why things go wrong when $\beta = 1$? What can you do to get a sensible answer?