Numerical Methods I

Problem Set 4

due in class, October 8, 2003

1. Let $\rho(A)$ denote the spectral radius of a matrix A, i.e.

$$\rho(A) = \max_{i=1,\dots,m} |\lambda_i| ,$$

where $\lambda_1, \dots, \lambda_m$ are the eigenvalues of A.

   (a) Show that $\rho(A) \leq \|A\|$ for any matrix norm of A.

   (b) Give an example of a nonzero matrix for which $\rho(A) = 0$.

2. Consider the Jacobi method for solving the equation $A\boldsymbol{x} = \boldsymbol{b}$, where

$$A = \begin{pmatrix} 1 & \varepsilon \\ \varepsilon & 1 \end{pmatrix}$$

For which values of ε does the method converge?

3. **Project:** Modify your program for the LU decomposition to include pivoting. I.e., let your program find P, L, and R so that for a given nonsingular matrix $A \in \mathbb{R}^{n \times n}$, $PA = LU$. Use your program to solve last week's Vandermonde test problem. Does pivoting help?

4. **Project:** Use LU decomposition with and without pivoting to solve randomly generated linear equations. Print the 2-norm of the residual and the condition number of the matrix for many test problems. Describe the effect of pivoting.

5. **Project:** Write an Octave code to perform a QR decomposition of an $n \times m$ matrix. Then compute the least squares solution to $A\boldsymbol{x} = \boldsymbol{b}$ with

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}, \qquad \boldsymbol{b} = \begin{pmatrix} 4 \\ 3 \\ 2 \\ 1 \end{pmatrix}.$$