Engineering and Science Mathematics at IUB

— Preparation Pack* (Version 1.0) —

June 28, 2002

1 Introduction

All students coming to the School of Engineering and Science at International University Bremen will have to take the first-year course *Engineering and Science Mathematics*. This document contains a number of typical problems which either reflect the material that a student should already be able to master upon arrival at IUB, or which will be addressed in the first semester of the course.

The material addressed in section 2 is expected to be known. If some of the material is new or if you are uncertain about it, please consult a suitable mathematics textbook. Section 3 deals with material that will be covered during the first semester of this course. Nonetheless, if all or most of it is new to you, we recommend to prepare as many topical fields as possible because the presentation must be rather concise to cover all the material needed in science and engineering in later semesters. If you have already mastered all the material, consider this as a set of useful revision problems.

However, you should not worry too much if you have difficulties with some problems. We will offer additional tutorials in the first year for all those who have deficiencies in one field or the other.

You find a questionnaire at the end of this set of problems. Please fill it in (anonymously, if you wish) in order to help us assessing the general level of prior knowledge of our students. The questionnaires will be collected in the first lecture of the course *Engineering and Science Mathematics*.

*The E&S Math Preparation Pack was compiled by the friendly GM2 team: *Joachim Vogt, Angela Köhler, Dietrich Burde, Michael Kropf.*

2 Basics (what you should know well)

1. Systems of two linear algebraic equations

Compute the solutions of the following sets of equations.

$$\text{(a)} \quad \begin{array}{rcl} 3x+y &=& 3 \\ x-y &=& 5 \end{array} \;, \qquad \text{(b)} \quad \begin{array}{rcl} 2x-y &=& 0 \\ -4x+2y &=& 1 \end{array} \;.$$

2. Quadratic equations

Find the roots of

$$\text{(a)} \quad x^2-6x+5 \;, \qquad \text{(b)} \quad x^2+x-1 \;, \qquad \text{(c)} \quad x^3-2x+1 \;.$$

[Hint for (c): divide by $(x-1)$.] Furthermore, factorize

$$\text{(d)} \quad x^2+4x+3 \;, \qquad \text{(e)} \quad a^2-6ab+9b^2 \;.$$

3. Partial fraction decomposition

Express the following in partial fractions.

$$\text{(a)} \quad \frac{x-3}{(x+1)(x-1)} \qquad \text{(b)} \quad \frac{6}{(x^2+2)(x-2)}$$

4. Exponents

Simplify the following expressions as far as possible.

$$\text{(a)} \quad 3^{1/2}3^{-1/3} \qquad \text{(b)} \quad a^{1/2}a^{-1/2} \qquad \text{(c)} \quad a^{1/2}+a^{-1/2} \qquad \text{(d)} \quad \frac{x^{-1/7}\cdot\left(x^{1/3}\right)^6}{x\cdot\sqrt[7]{x^{-1}}\cdot\sqrt{x}}$$

5. Logarithms

Simplify the following expressions as far as possible ($\ln \equiv \log_e$).

$$\text{(a)} \quad 2\ln\sqrt{2} \qquad \text{(b)} \quad e^{-\ln x}$$

Write the following expressions in terms of $\ln a$ and $\ln b$, if possible.

$$\text{(c)} \quad \ln(a+b) \qquad \text{(d)} \quad \ln\frac{a}{b}$$

6. Trigonometric functions

By writing $\pi/12 = \pi/3 - \pi/4$, use trigonometric identities to evaluate

$$\text{(a)} \quad \cos\frac{\pi}{12}\,, \qquad \text{(b)} \quad \sin\frac{\pi}{12}\,, \qquad \text{(c)} \quad \tan\frac{\pi}{12}\,.$$

7. Triangles

Look at the following triangles ABC and determine all sides and angles that are not given:

$$\text{(a)} \quad AB = 2,\ BC = 2,\ A = \pi/3 \text{ radians}\,, \qquad \text{(b)} \quad AB = 2,\ BC = 2,\ AC = 3\,.$$

8. Minimax problems

Compute the derivatives of the following functions, find the stationary points, and determine whether they are local maxima, local minima or points of inflexion.

$$\text{(a)} \quad y = 2x^2 - 1 \qquad \text{(b)} \quad y = x^3 - 6x^2 + 12x + 3 \qquad \text{(c)} \quad y = x^3 - 12x + 3$$

9. Derivatives

Compute the derivatives of the following functions $y = y(x)$.

$$\text{(a)} \quad y = \cos(x^3) \qquad \text{(b)} \quad y = 2^x \qquad \text{(c)} \quad y = \ln(x^2 + 2) \qquad \text{(d)} \quad y = \arcsin(x^x)$$

10. Elementary integrals

Compute the following integrals.

$$\text{(a)} \quad \int (3x^2 - 5)\,\mathrm{d}x \qquad \text{(b)} \quad \int_1^8 \sqrt[3]{x}\,\mathrm{d}x \qquad \text{(c)} \quad \int \cos(2x)\,\mathrm{d}x \qquad \text{(d)} \quad \int 5\,\mathrm{e}^{-3x+8}\,\mathrm{d}x$$

11. Transformations of a function

Let $f(x) = x^3$. By considering transformations on f, sketch the graphs of the following functions.

$$\text{(a)} \quad y = f(x) \qquad \text{(b)} \quad y = -2f(x) + 1 \qquad \text{(c)} \quad y = f(x - 1) \qquad \text{(d)} \quad y = f(3x + 1) - 2$$

12. Solutions of equations

(a) If $2 = 8^{-x}$ find x.

(b) Find all values of α in the range 0 to 2π which satisfy the equation $4\cos^2\alpha = 3$.

(c) If $\log_2 x = 2\log_{10} 7x$ find x.

13. Graphs of elementary functions

Sketch the graphs of the following functions, for $-\infty < x < \infty$.

(a) $y = \sin 2x$ (b) $y = 2\tan x$ (c) $y = (\cos x)^2$ (d) $y = \mathrm{e}^{-x}\sin x$

14. Parametric curves

Sketch the curves in the xy plane given by

(a) $x = t + 1,\ y = 2t + 1,$ (b) $x = t^2 + 1,\ y = 2t^2 + 1,$

as well as

(c) $x = 3\cos t,\ y = 2\sin t,$ (d) $x = \mathrm{e}^{-t^2}\cos t,\ y = \mathrm{e}^{-t^2}\sin t,$

where $-\infty < t < \infty$.

15. Everyday mathematics

(a) How much time does a photon (speed $c = 300,000$ km/sec) need to travel 10 cm ? Give the answer with correct units.

(b) A box has length 10 mm, width 10 mm, and a volume of 10 ml. What is its height?

(c) Two NaCl solutions are given, one with 1 g salt per liter, the other one with 0.45 g salt per liter. In what quantities do you need to mix them to get exactly ten liters of solution with a salt concentration of 0.6 g/l ?

3 Material covered in the first semester

16. Vector operations

Consider the three vectors

$$\mathbf{u} = \begin{pmatrix} 1 \\ 8 \\ -4 \end{pmatrix} , \quad \mathbf{v} = \begin{pmatrix} 6 \\ 8 \\ 0 \end{pmatrix} , \quad \mathbf{w} = \begin{pmatrix} 4 \\ -4 \\ -7 \end{pmatrix} .$$

(a) Order the vectors by magnitude.

(b) Find the angles between all pairs of vectors.

(c) Determine the area of the triangle given by the origin and the endpoints of $\mathbf{u}$ and $\mathbf{v}$.

(d) Determine the volume of the parallelepiped spanned by the three vectors.

17. Angles between lines and planes

Find the angle between

(a) the line $x = y = -z$ and the plane $z = 0$,

(b) the planes $y = 0$ and $x + y - 1 = 0$,

(c) the lines $x = y = -z$ and $x = -y = 2z$.

18. Intersection point

Find the point of intersection of the plane $\mathcal{P}$ and the line $\mathcal{L}$ given by

$$\mathcal{P} : \begin{pmatrix} 2 \\ 5 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} , \quad \mathcal{L} : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \nu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} .$$

19. Basic notions from set theory

Consider the two sets $A = \{1, 2, 4\}$ and $B = \{2, 3, 4, 5\}$. Write down

$$\text{(a)} \quad A \cup B , \qquad \text{(b)} \quad A \cap B , \qquad \text{(c)} \quad C = \{x \in B : x \notin A\} .$$

20. Sequences and series

Evaluate (a) the sum of the odd integers from 5 to 49 inclusive, and

$$\text{(b)} \quad \sum_{n=1}^{10} (2n + 5) , \qquad \text{(c)} \quad \sum_{n=0}^{N} (an + b) , \qquad \text{(d)} \quad \sum_{n=0}^{10} 2^n , \qquad \text{(e)} \quad \sum_{n=0}^{N} a\, c^{2n} .$$

(a, b, c, and N are constants.)

21. Basic notions from probability theory

Let X and Y be two independent random quantities with two possible outcomes X_1, X_2, and Y_1, Y_2, respectively. Suppose the probablities are given by $P(X = X_1) = 0.3$ and $P(Y = Y_2) = 0.6$. Compute the probabilities

(a) $P(X = X_2)$, (b) $P(X = X_1 \text{ and } Y = Y_2)$, (c) $P(X = X_1 \text{ and not } Y = Y_2)$.

22. Matrix calculus

The matrices $\mathsf{A}, \mathsf{B}, \mathsf{C}$ are given by

$$\mathsf{A} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} , \qquad \mathsf{B} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 0 & -1 \end{pmatrix} , \qquad \mathsf{C} = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \end{pmatrix} .$$

Which of the following matrices are defined:

$$\mathsf{B} + \mathsf{C} \ , \ \mathsf{A} - \mathsf{B} \ , \ \mathsf{CB} \ , \ \mathsf{BC} \ , \ \mathsf{AA}^t \ , \ \mathsf{B}^2 \ , \ \mathsf{A}^{-1} \ , \ \mathsf{B}^{-1} \ .$$

Note that M^t and M^{-1} denote the transpose and the inverse of a matrix M, respectively. Compute those matrices which are defined.

23. Methods of integration

Compute the following integrals.

$$\text{(a)} \quad \int x\mathrm{e}^x \,\mathrm{d}x \qquad \text{(b)} \quad \int \frac{\sin\sqrt{x}}{\sqrt{x}} \,\mathrm{d}x$$

24. System of linear equations

Compute the solution of the following system of linear equations.

$$\begin{aligned} a + b + c + d &= 1 \\ a - b + c - d &= 1 \\ 2a + b - c - 2d &= -1 \\ 2a - b - c - 2d &= 1 \end{aligned}$$

25. Implicit differentiation

Compute the derivatives $y'(x) \equiv \mathrm{d}y/\mathrm{d}x$ of the implicitly given functions $y = y(x)$.

(a) $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$ at the point $\left(\dfrac{6}{5}, \dfrac{12}{5}\right)$

(b) $\mathrm{e}^{x-y} + x^2 + y^2 = 3$ at the point $(1, 1)$

4 Answers

1. Systems of two linear algebraic equations

(a) $x = 2,\ y = -3$ (b) no solution

2. Quadratic equations

(a) $x_1 = 1,\ x_2 = 5$ (b) $x_{1,2} = \dfrac{-1 \pm \sqrt{5}}{2}$ (c) $x_1 = 1,\ x_{2,3} = \dfrac{-1 \pm \sqrt{5}}{2}$

(d) $x^2 + 4x + 3 = (x+3)(x+1)$ (e) $a^2 - 6ab + 9b^2 = (a - 3b)^2$

3. Partial fraction decomposition

(a) $\dfrac{2}{x+1} - \dfrac{1}{x-1}$ (b) $\dfrac{1}{x-2} - \dfrac{x+2}{x^2+2}$

4. Exponents

(a) $3^{1/6} \equiv \sqrt[6]{3}$ (b) $a^0 = 1$ (c) $a^{-1/2}(a+1)$ (d) $x^{1/2} \equiv \sqrt{x}$

5. Logarithms

(a) $\ln 2$ (b) $1/x$ (c) not possible (d) $\ln a - \ln b$

6. Trigonometric functions

(a) $\dfrac{\sqrt{6}+\sqrt{2}}{4}$ (b) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$ (c) $\dfrac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}} = 2 - \sqrt{3}$

7. Triangles

(a) $AC = 2,\ B = C = \pi/3$ radians (b) $A = \arccos\dfrac{3}{4},\ B = \arccos\left(-\dfrac{1}{8}\right),\ C = \arccos\dfrac{3}{4}$

8. Minimax problems

(a) $x = 0$: local minimum

(b) $x = 2$: inflexion point

(c) $x = 2$: local minimum, $x = -2$: local maximum

9. Derivatives

(a) $-3x^2\sin(x^3)$ (b) $(\ln 2)\,\mathrm{e}^{x\ln 2} \equiv (\ln 2)\,2^x$ (c) $\dfrac{2x}{x^2+2}$ (d) $\dfrac{x^x(\ln x+1)}{\sqrt{1-x^{2x}}}$

10. Elementary integrals

(a) x^3-5x+C (b) $45/4$ (c) $\dfrac{1}{2}\sin 2x\,+C$ (d) $-\dfrac{5}{3}\,\mathrm{e}^{-3x+8}+C$

12. Solutions of equations

(a) $x=-1/3$.

(b) $x=\pi/6,\ 5\pi/6,\ 7\pi/6,\ 11\pi/6$.

(c) $x=7^{\alpha/(1-\alpha)}$ where $\alpha = 2\log_{10}2=\log_{10}4$.

15. Everyday mathematics

(a) $(1/3)\,10^{-9}$ sec (= one third of a nanosecond)

(b) 100 mm

(c) 30/11 liter of the first solution plus 80/11 liter of the second solution

16. Vector operations

(a) $|\mathbf{v}| > |\mathbf{u}| = |\mathbf{w}|$

(b) $\angle(\mathbf{u},\mathbf{v}) = \arccos(7/9)$, $\angle(\mathbf{u},\mathbf{w}) = \arccos 0 = \pi/2$, $\angle(\mathbf{v},\mathbf{w}) = \arccos(-4/45)$

(c) $20\sqrt{2}$

(d) 504

17. Angles between lines and planes

(a) $\arccos(\sqrt{6}/3)$ (b) $\arccos(\sqrt{2}/2)=\pi/4$ (c) $\arccos[1/(3\sqrt{3})]$

18. Intersection point

$(3,0,2)$

19. Basic notions from set theory

(a) $A\cup B=\{1,2,3,4,5\}$ (b) $A\cap B=\{2,4\}$ (c) $C=\{3,5\}$

20. Sequences and series

$$\text{(a) } 621 \quad \text{(b) } 160 \quad \text{(c) } (b + aN/2)(N+1) \quad \text{(d) } 2^{11} - 1 \quad \text{(e) } a\,\frac{c^{2(N+1)-1}}{c^2 - 1}$$

21. Basic notions from probability theory

$$\text{(a)} \quad 0.7 \qquad \text{(b)} \quad 0.18 \qquad \text{(c)} \quad 0.12$$

22. Matrix calculus

$$\mathsf{CB} = \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}, \qquad \mathsf{BC} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & -1 \\ 0 & -1 & 0 \end{pmatrix}, \qquad \mathsf{AA}^t = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

that means,

$$\mathsf{A}^{-1} = \mathsf{A}^t = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$

The other matrices are not defined.

23. Methods of integration

$$\text{(a)} \quad (x-1)\,\mathrm{e}^x + C \qquad \text{(b)} \quad -2\cos\sqrt{x} + C$$

24. System of linear equations

$a = 1,\ b = -1,\ c = 0,\ d = 1$

25. Implicit differentiation

(a) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{9x}{4y} = -\dfrac{9}{8}$

(b) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{2x + \mathrm{e}^{x-y}}{2y - \mathrm{e}^{x-y}} = -3$

5 Questionnaire

Please tick one box in each table for each question.

Material covered at school		
	Yes	No
Qn. 1		
Qn. 2		
Qn. 3		
Qn. 4		
Qn. 5		
Qn. 6		
Qn. 7		
Qn. 8		
Qn. 9		
Qn. 10		
Qn. 11		
Qn. 12		
Qn. 13		
Qn. 14		
Qn. 15		
Qn. 16		
Qn. 17		
Qn. 18		
Qn. 19		
Qn. 20		
Qn. 21		
Qn. 22		
Qn. 23		
Qn. 24		
Qn. 25		

Difficulty of question					
	Easy				Difficult
Qn. 1					
Qn. 2					
Qn. 3					
Qn. 4					
Qn. 5					
Qn. 6					
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