Engineering and Science Mathematics I

Final Exam – Standard Track

December 21, 2002

1. Find the following limits. (Some of these limits may not exist!)

   (a) $\displaystyle\lim_{x\to 1^-} \frac{|x-1|}{x-1}$

   (b) $\displaystyle\lim_{x\to 0} \sqrt{x}\,\ln x$

   (c) $\displaystyle\lim_{n\to\infty} 1 + \left(\frac{3}{2}\right)^n$

   (d) $\displaystyle\lim_{n\to\infty} \left(\frac{1+n}{n}\right)^n$

   $(5+5+5+5)$

2. The radius of a circle is measured with a relative error of 3%. Give an estimate for the relative error in the computed area of the circle. (10)

3. Find the point on the line
$$\boldsymbol{l} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$
   that is closest to the origin.

   Hint: Find the minimum of $f(\lambda) = |\boldsymbol{l}|^2$. (10)

4. Let $f(x) = x^{1/3}$.

   (a) Compute the first three non-zero terms of the Taylor series of f about the point $a = 1$.

   (b) Find the equation for the line tangent to f at the point $x = 1$.

   $(10+10)$

5. Consider the function
$$f(x) = \arctan x\,.$$
   Find the domain, intercepts, horizontal and vertical asymptotes, extrema and inflection points of f. Identify all minima and maxima of f, as well regions where the graph is concave upward or concave downward. Finally sketch the graph into the coordinate system provided. (20)

6. Compute the following integrals.

   (a) $\displaystyle\int \frac{\sec x}{\tan x}\, dx$

   (b) $\displaystyle\int x^2 \cos^2 x\, dx$

   (c) $\displaystyle\int \frac{x^2}{\sqrt{4-x^2}}\, dx$

   (d) $\displaystyle\int \frac{\sin x}{\sin 2x}\, dx$

   $(10+10+10+10)$

7. Compute the area of the surface generated by revolving the curve

   $$y = \frac{1}{3}x^3, \qquad 0 \le x \le 1,$$

   about the x-axis. $\hfill (10)$

8. Determine if the following definite integrals are zero, positive, or negative. You do not need to evalute any of them, but a short explanation of your reasoning is required!

   (a) $\displaystyle\int_{-1}^{1} \left(x + x^3 + x^5 + x^7 + x^9 + x^{11} + x^{13}\right) dx$

   (b) $\displaystyle\int_{0}^{\pi} \left(\sin^{99} x - \sin x\right) dx$

   (c) $\displaystyle\int_{-5}^{5} \left(e^x - 1\right) dx$

   (d) $\displaystyle\int_{0}^{2\pi} \left(\sin^{100} x - \cos^{100} x\right) dx$

   $(5+5+5+5)$

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