110231 Nonlinear Dynamics Lab

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Essentials on 2-dimensional (and n-dimensional) ODEs

$$\boxed{\frac{dx}{dt} = 5x + y, \quad \frac{dy}{dt} = -x - y}$$

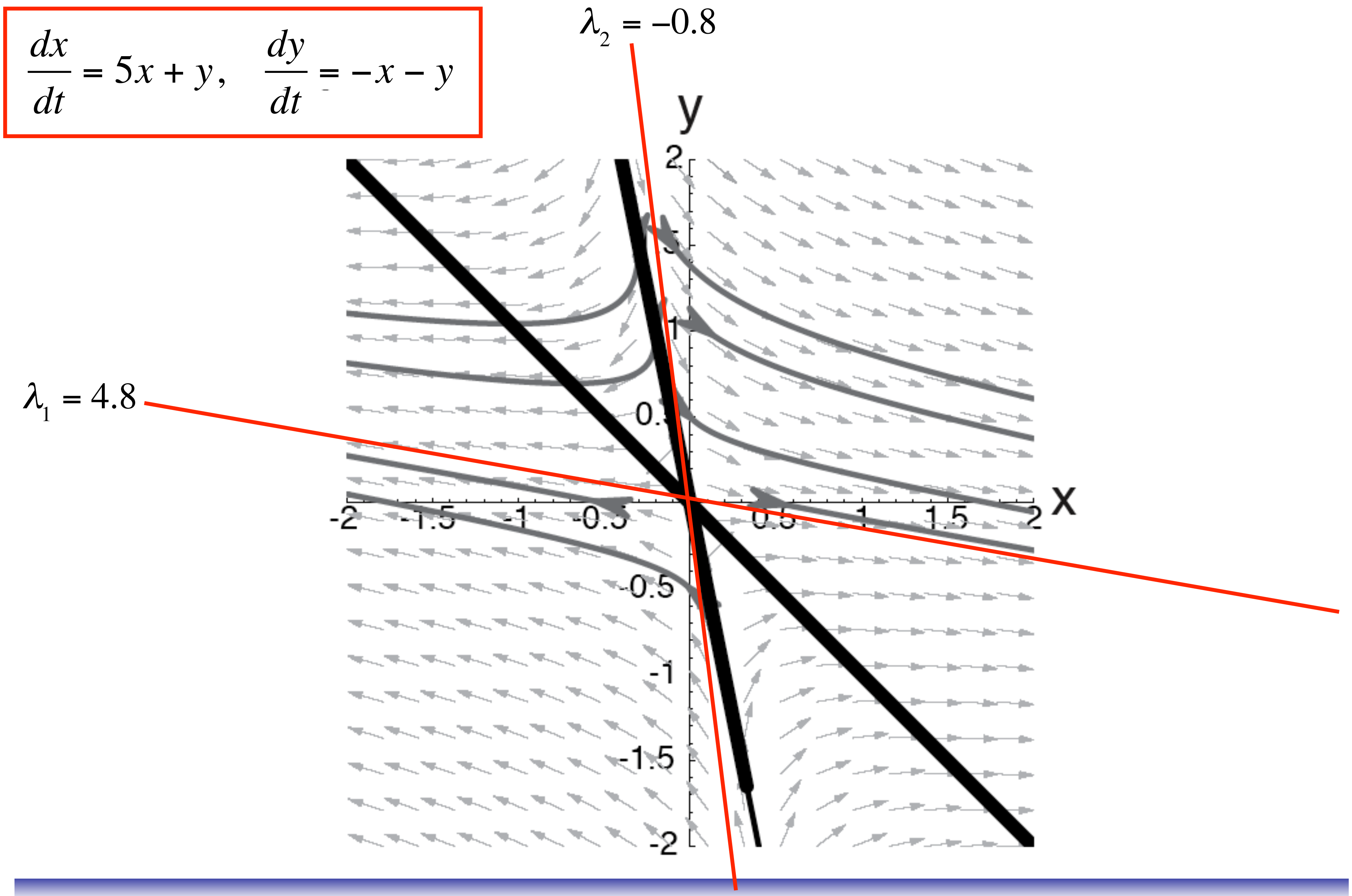
$$A = \begin{pmatrix} 5 & 1 \\ -1 & -1 \end{pmatrix} \longrightarrow$$

eigenvalues $\lambda_1 = 4.8, \quad \lambda_2 = -0.8$

eigenvectors $\vec{v}_1 = \begin{pmatrix} -5.8 \\ 1 \end{pmatrix}, \quad \vec{v}_2 = \begin{pmatrix} -0.2 \\ 1 \end{pmatrix}$

Essentials on 2-dimensional (and n-dimensional) ODEs

$$\frac{dx}{dt} = 5x + y, \quad \frac{dy}{dt} = -x - y$$



Essentials on 2-dimensional (and n-dimensional) ODEs

$$\frac{d\vec{x}}{dt} = A\vec{x}$$

- **transfer of methods to general 2-dim ODE** $\frac{dx}{dt} = f(x,y), \quad \frac{dy}{dt} = g(x,y)$
- **linearization around a fixed point** (x_F, y_F)
- **Jacobian matrix** $J = \left.\begin{pmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{pmatrix}\right|_{(x,y)=(x_F,y_F)}$

→ **same machinery as before**

Essentials on 2-dimensional (and n-dimensional) ODEs

$$\boxed{\frac{d\vec{x}}{dt} = A\vec{x}}$$

$$\rightarrow \quad \vec{x}(t) = e^{\lambda t}\vec{v}$$

$$\lambda = \mathrm{Re}\,\lambda + i\,\mathrm{Im}\,\lambda$$

$$\rightarrow \quad e^{\lambda t} = e^{\mathrm{Re}\,\lambda t + i\,\mathrm{Im}\,\lambda t} = e^{\mathrm{Re}\,\lambda t}e^{i\,\mathrm{Im}\,\lambda t}$$

$$\rightarrow \quad \vec{x}(t) = e^{\mathrm{Re}\,\lambda t}e^{i\,\mathrm{Im}\,\lambda t}\vec{v}$$

exponential
growth/decay

oscillatory contribution

Glycolysis

- biochemical pathway for sugar degradation
- key process in "energy management"

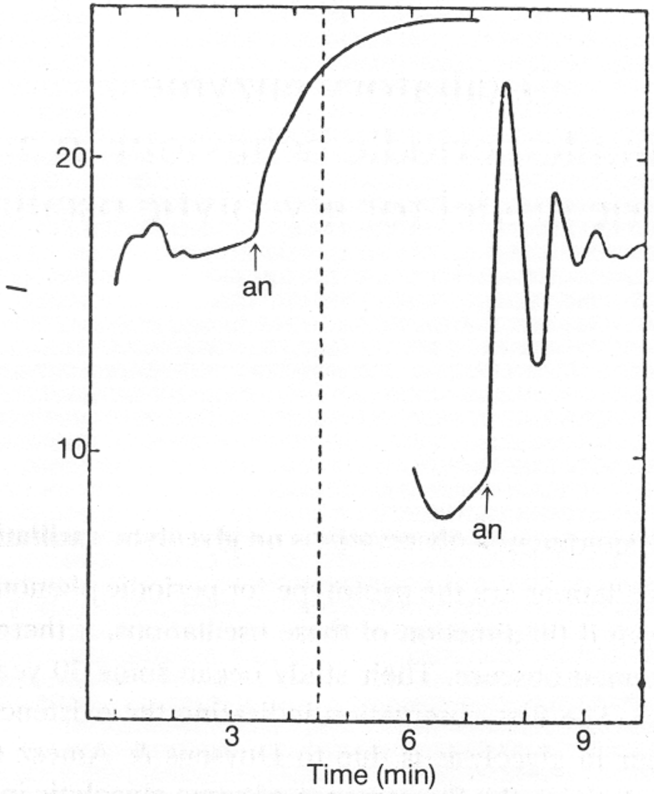


Fig. 2.1. Damped oscillations in the fluorescence of a glycolytic intermediate, NADH, following the injection of glucose (right) in a suspension of yeast cells. This observation was the first indication of the possibility of oscillatory behaviour in glycolysis. The curve on the left shows the addition of ethanol. an, anaerobic condition (Duysens & Amesz, 1957).

A. Goldbeter, *Biochemical oscillations and cellular rhythms*. Cambridge Univ.Press, 1996

Glycolysis

- biochemical pathway for sugar degradation
- key process in “energy management”

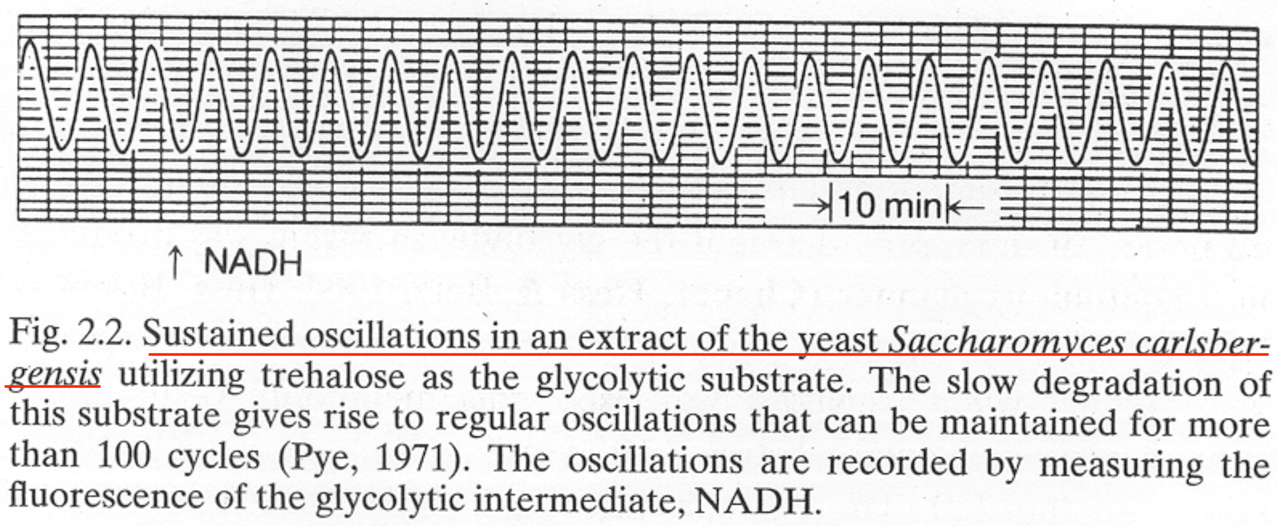
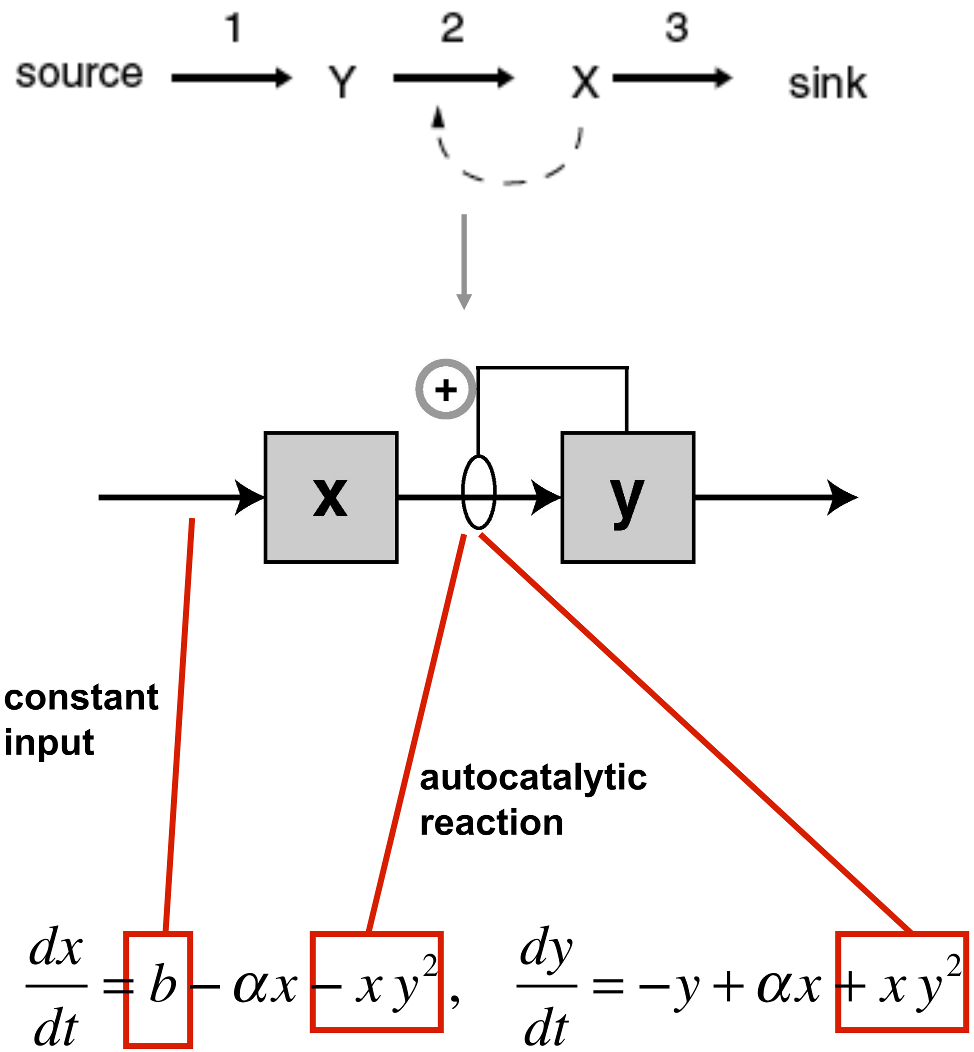


Fig. 2.2. Sustained oscillations in an extract of the yeast *Saccharomyces carlsbergensis* utilizing trehalose as the glycolytic substrate. The slow degradation of this substrate gives rise to regular oscillations that can be maintained for more than 100 cycles (Pye, 1971). The oscillations are recorded by measuring the fluorescence of the glycolytic intermediate, NADH.

A. Goldbeter, *Biochemical oscillations and cellular rhythms*. Cambridge Univ.Press, 1996

Glycolysis



constant input

autocatalytic reaction

$$\frac{dx}{dt} = \boxed{b} - \alpha x \boxed{- x y^2}, \quad \frac{dy}{dt} = -y + \alpha x \boxed{+ x y^2}$$

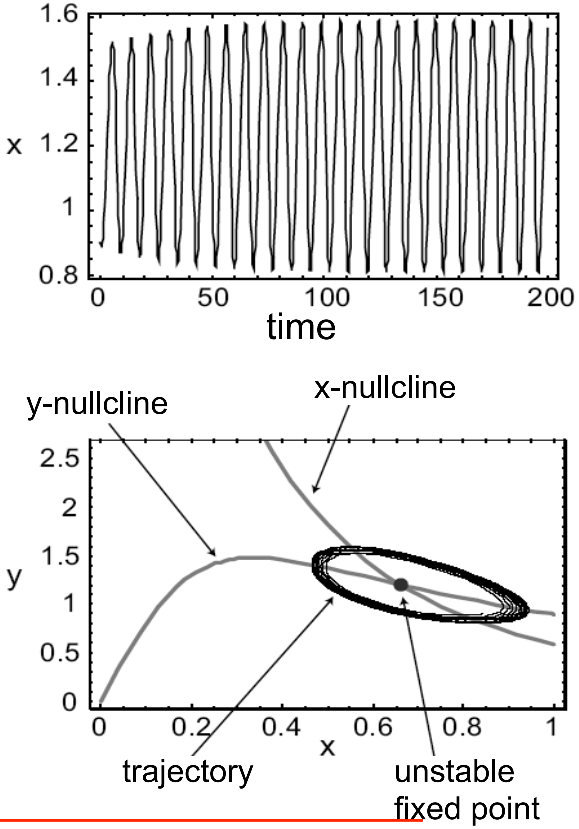
Sel'kov oscillator

Glycolysis

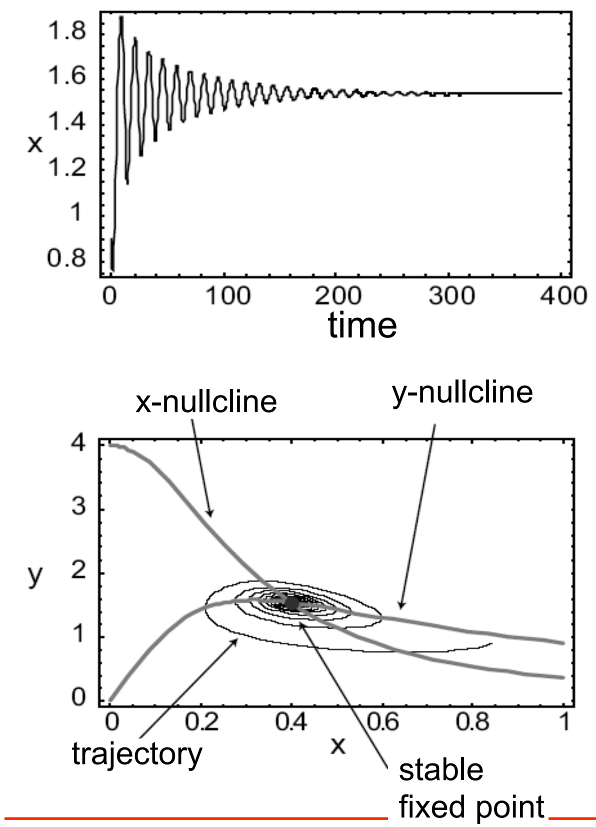
$$\frac{dx}{dt} = b - \alpha x - x\,y^2, \quad \frac{dy}{dt} = -y + \alpha x + x\,y^2$$

Sel’kov oscillator

parameter set 1:



parameter set 2:



Task:

- Select an example of two coupled linear ODEs and explore them using *Mathematica*
  - nullclines
  - numerical trajectories
  - eigenvectors and eigenvalues
- Analyze the Sel'kov oscillator:
  - compute the fixed point(s)
  - reproduce the two cases on the previous slide
  - compute the boundary line in parameter space between stable fixed point and stable oscillation