# 110231 Nonlinear Dynamics Lab

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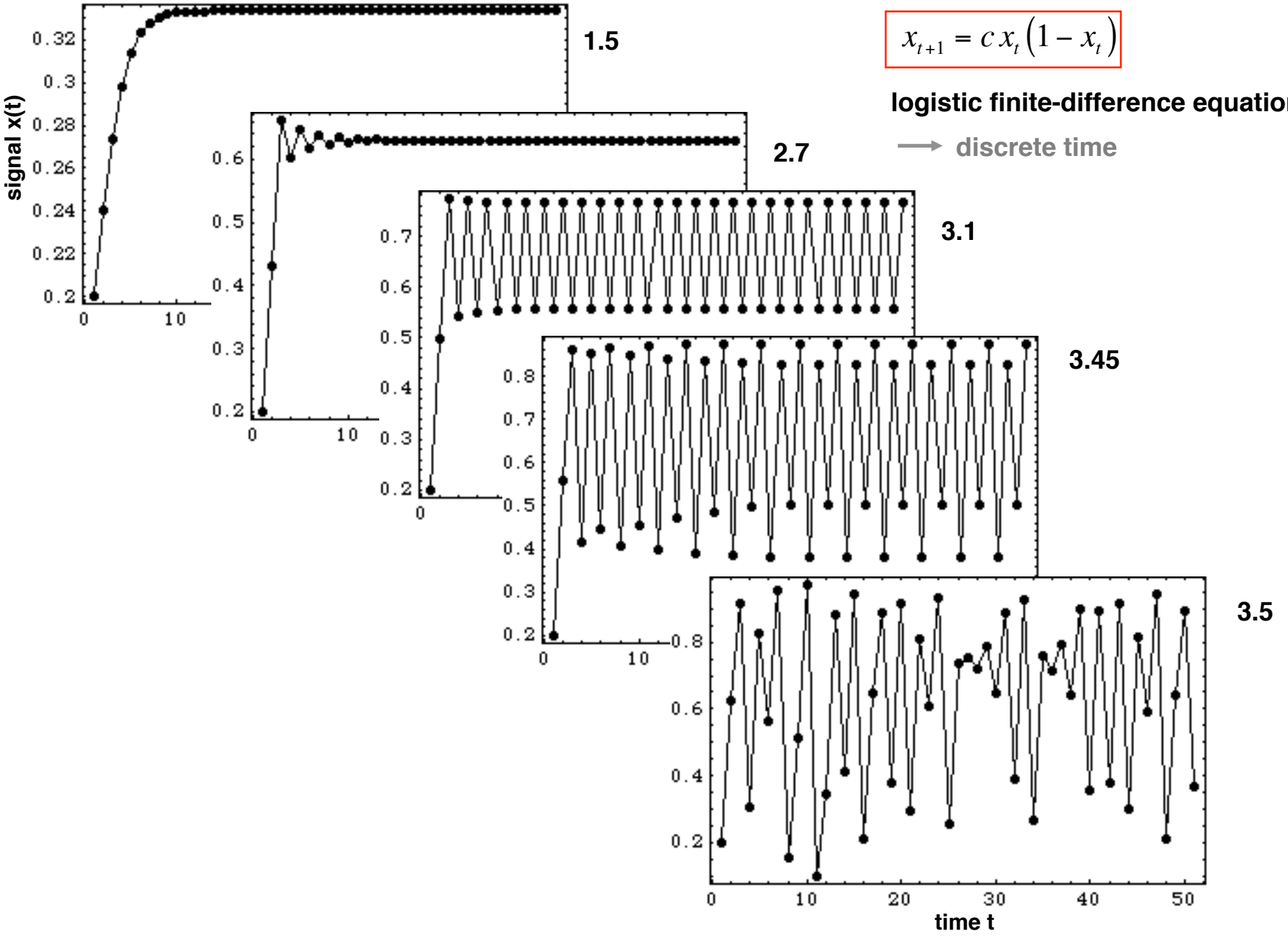


$$x_{t+1} = c\,x_t\left(1 - x_t\right)$$

**logistic finite-difference equation**

→ discrete time

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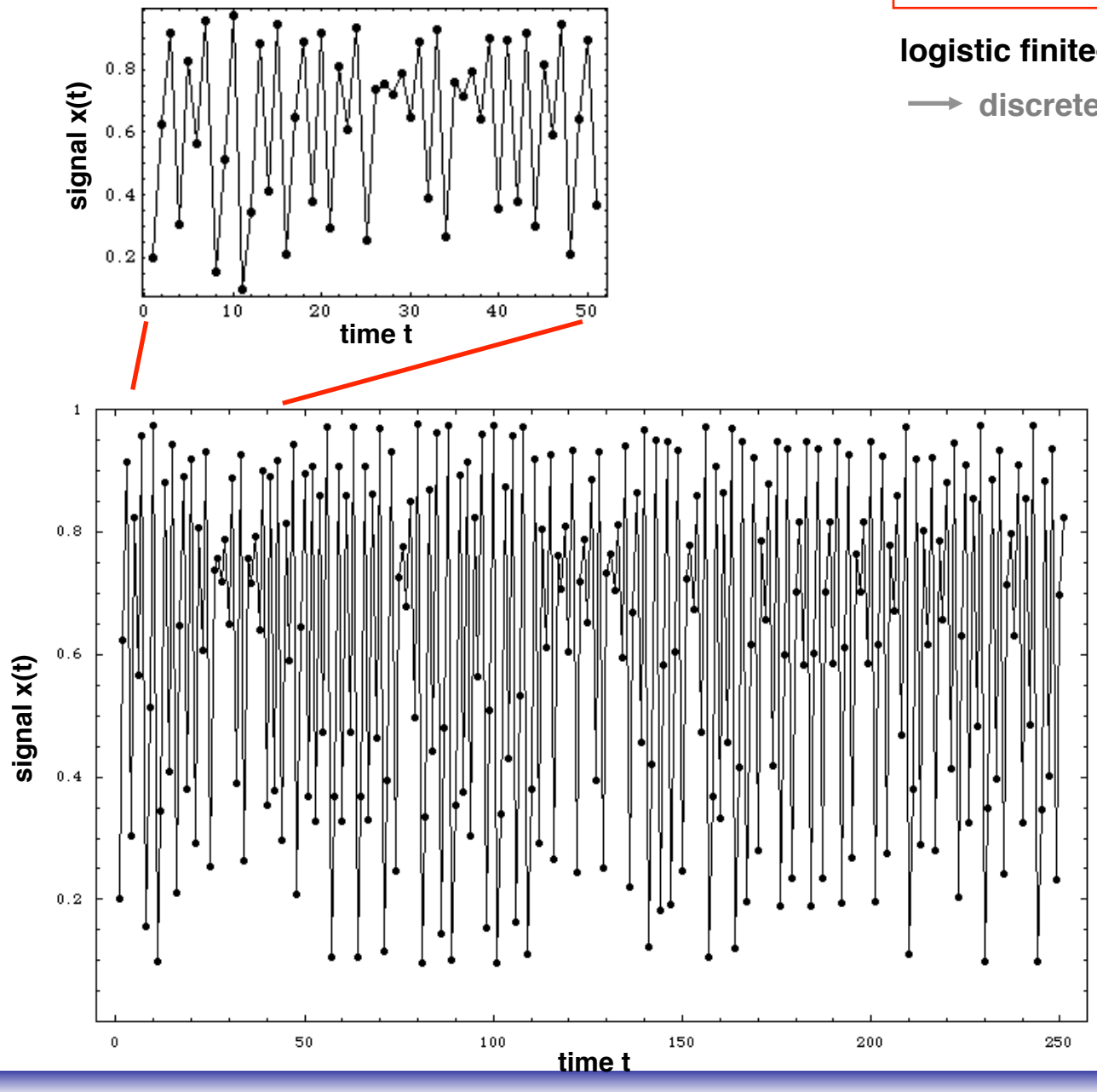
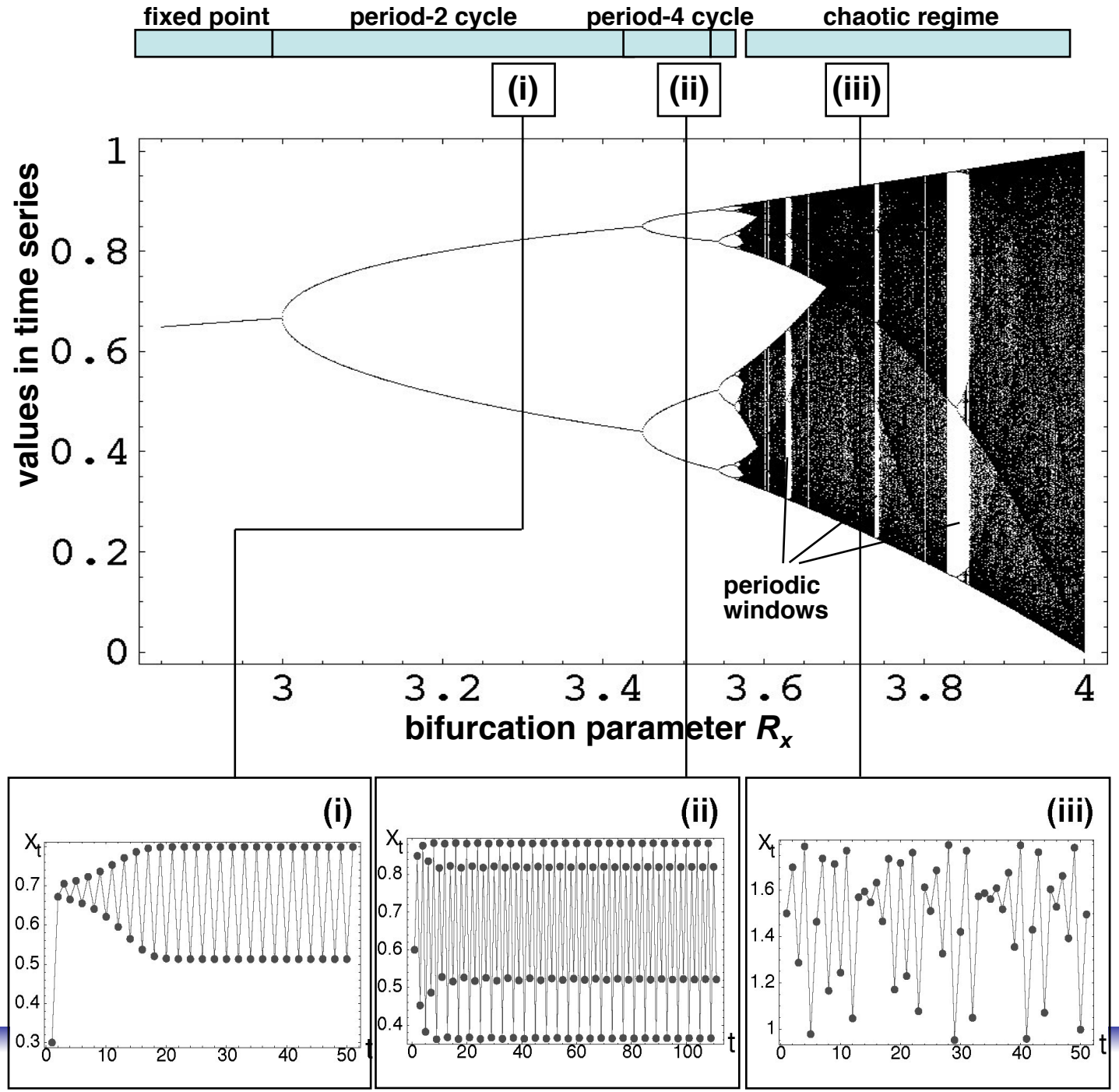


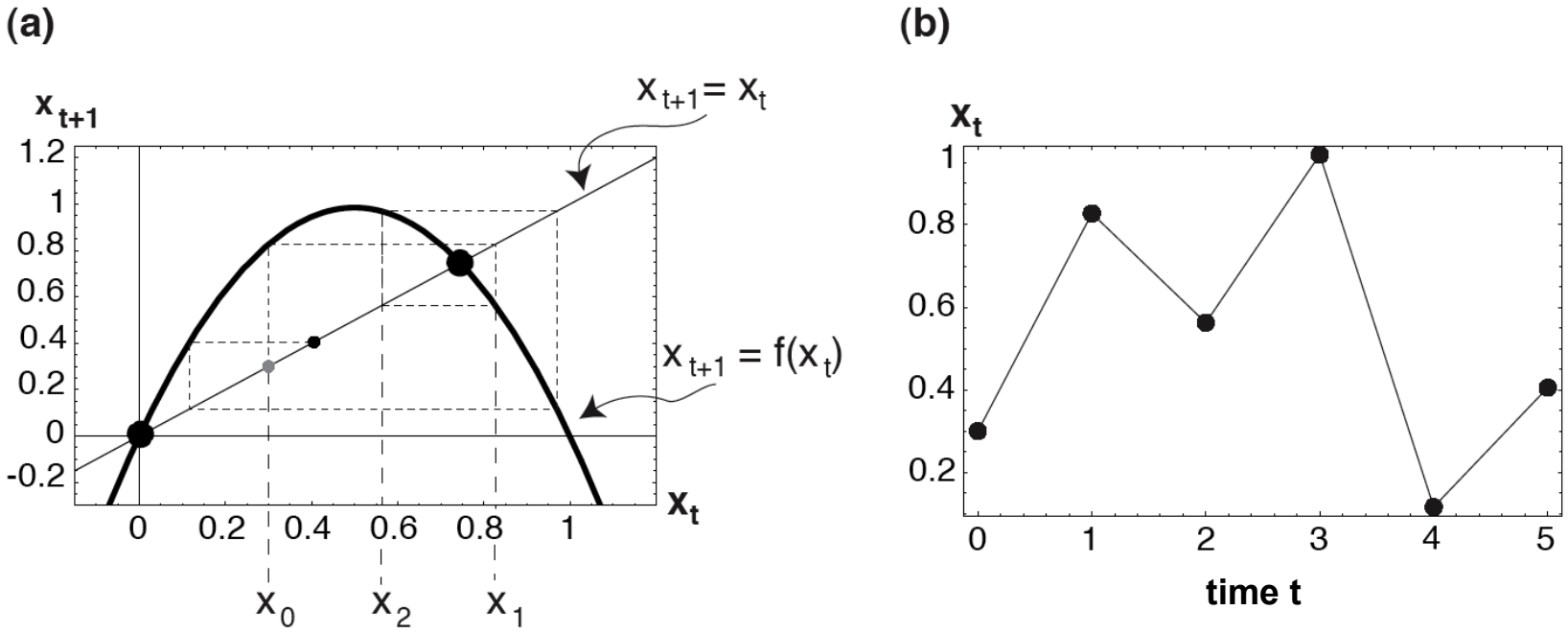

fixed point
period-2 cycle
period-4 cycle
chaotic regime
(i)
(ii)
(iii)
values in time series
1
0.8
0.6
0.4
0.2
0
3
3.2
3.4
3.6
3.8
4
bifurcation parameter $R_x$
periodic windows
(i)
$X_t$
0.7
0.6
0.5
0.4
0.3
0
10
20
30
40
50
t
(ii)
$X_t$
0.8
0.7
0.6
0.5
0.4
0
20
40
60
80
100
t
(iii)
$X_t$
1.6
1.4
1.2
1
0
10
20
30
40
50
t

**Roadmap for today (and the next two sessions):**

- implementing finite-difference equations in *Mathematica*
- studying the stability of fixed points and oscillations
- exploring ordinary differential equations in *Mathematica*

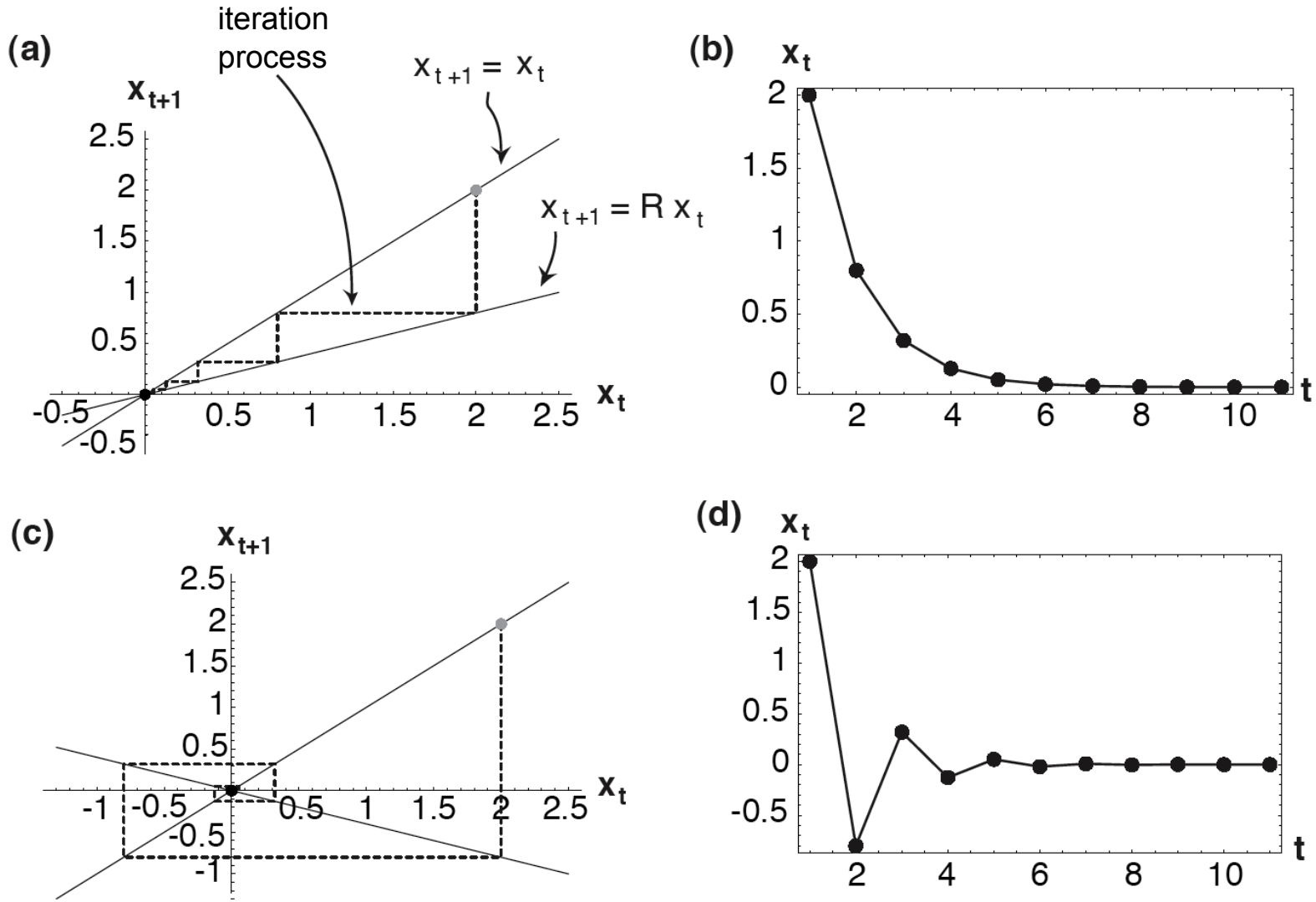
## Finite-difference equations

### graphical iteration scheme for finite-difference equations

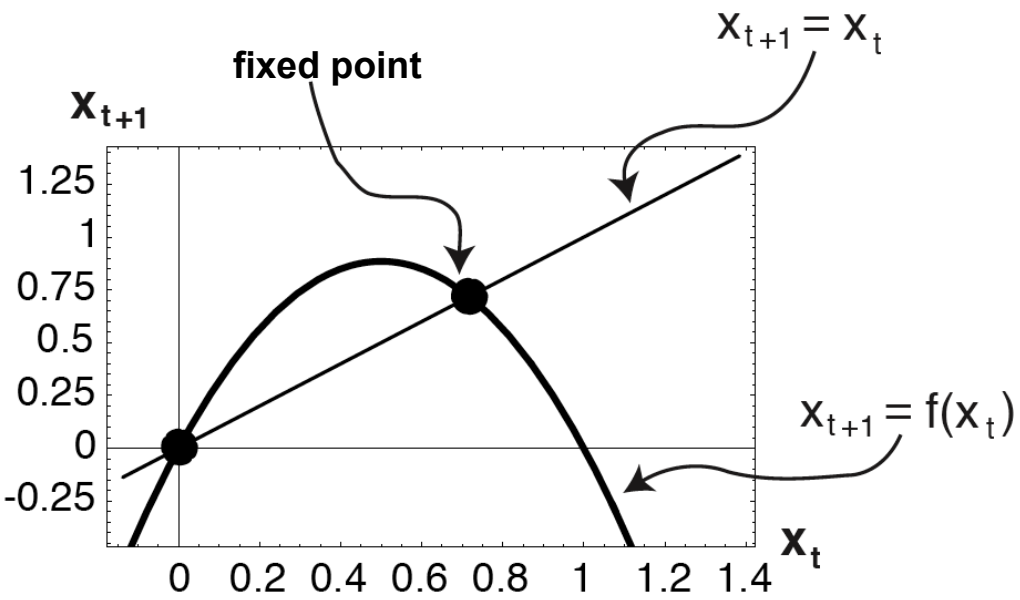


## Finite-difference equations

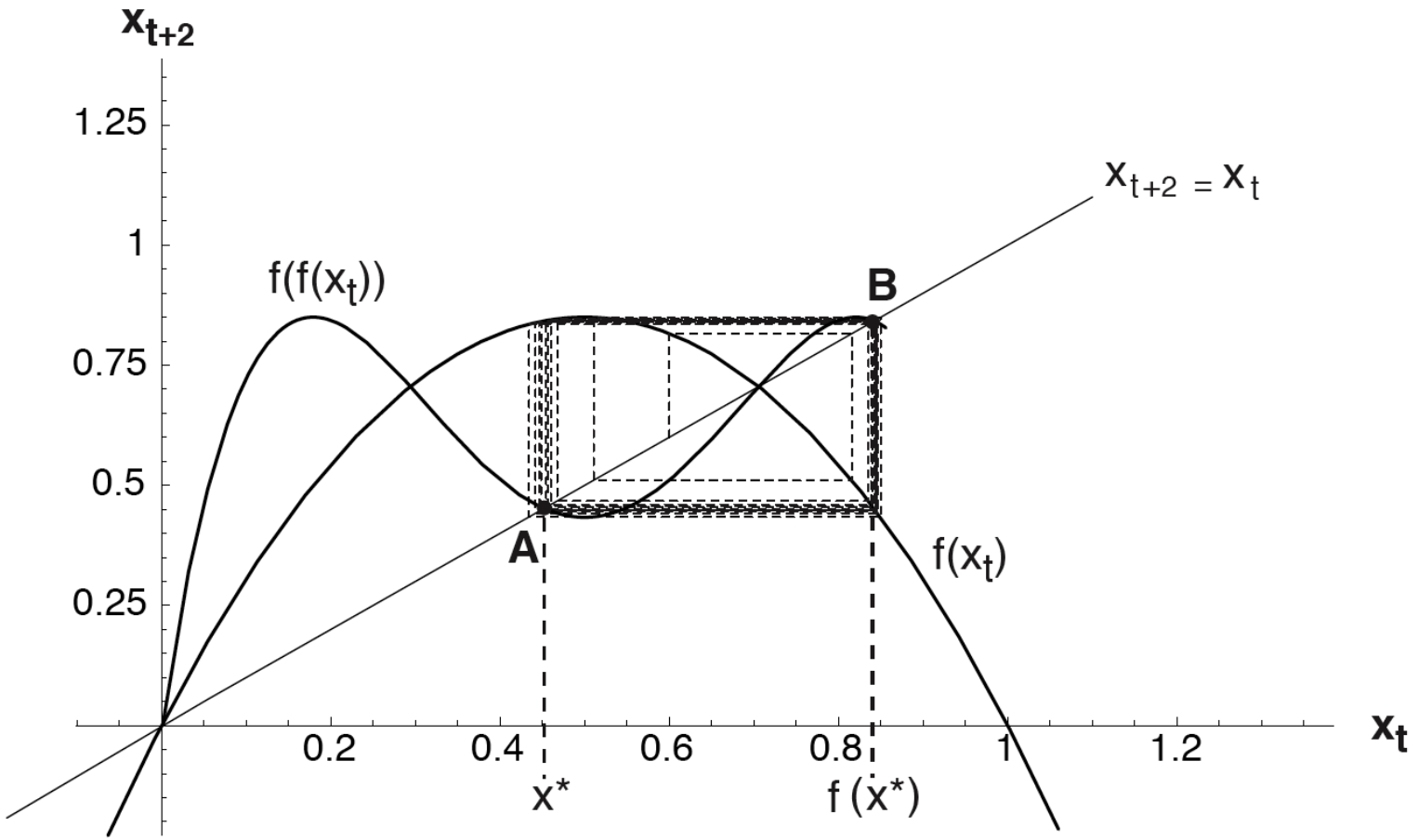
### linear finite-difference equations



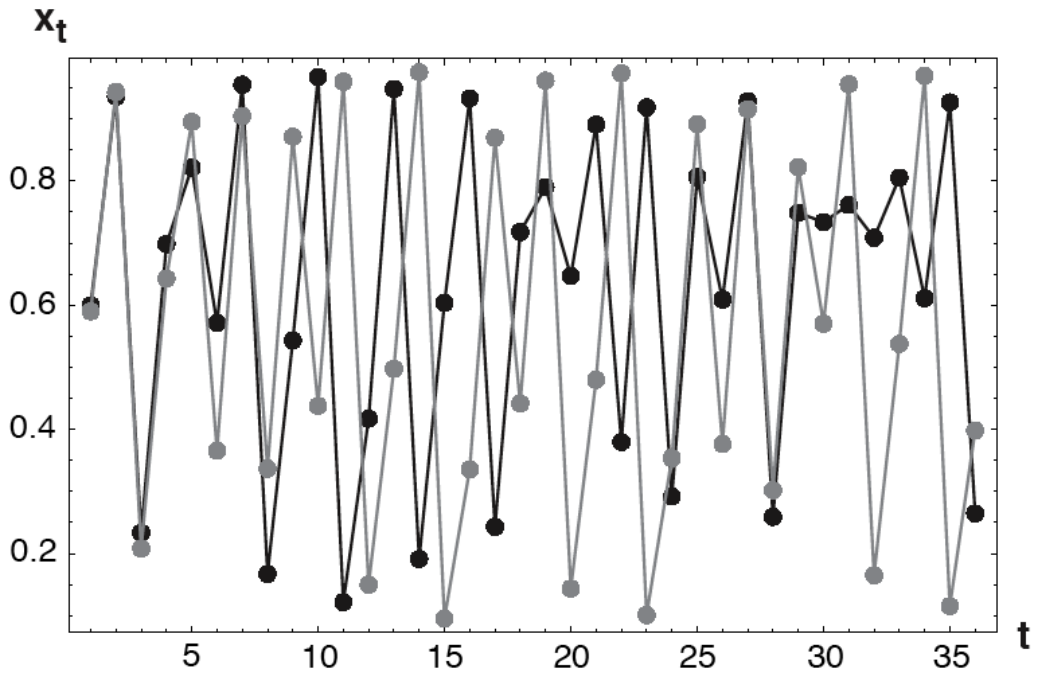
## Finite-difference equations



## Finite-difference equations



## Finite-difference equations



**First task:**

- Write a small program for iterating such maps
- Write a small program for displaying time series as a trajectory in a 'return plot' (i.e. an $(x_{t+1}, x_t)$ diagram).
- Use Mathematica to semi-analytically analyze the stability of the period-2 cycle